

Chapter 2 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 2: Differentiation Rules.

- 2.1: Defining a Derivative
- 2.2: Differentiating Power, Exponential, and Sinusoidal Functions
- 2.3: Product Rule and Quotient Rule
- 2.4: Chain Rule
- 2.5: Implicit Differentiation and Differentiating Inverse Functions
- 2.6: Differentiating Logarithmic Functions
- 2.7: Related Rates
- 2.8: Linearization and Differentials
- 2.9: Hyperbolic Functions

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

Visit valscalc.com to access a free online calculus textbook and additional multiple-choice and free-response practice exams.

Differentiation Rules**Number of Questions—50****NO CALCULATOR**

1. A tree's height as a function of time is modeled by $h(t)$, where $h(t)$ is measured in feet and t is measured in years since the tree was planted. What is the best interpretation of the equation $h'(5) = 2$?
- (A) When the tree is 2 years old, it is 5 feet tall.
- (B) When the tree is 2 years old, it is growing at a rate of 2 feet per year.
- (C) When the tree is 5 years old, it is 2 feet tall.
- (D) When the tree is 5 years old, it is growing at a rate of 2 feet per year.
- (E) When the tree is 5 years old, it is growing at a rate of 5 feet per year.

2. If $y = \sin(2x + 1)$, then $\frac{dy}{dx} =$

(A) $\cos(2x + 1)$

(B) $-2\cos(2x + 1)$

(C) $2\cos(2x + 1)$

(D) $2\sin(2x + 1)$

(E) $-2\sin(2x + 1)$

3. $\frac{d}{dx} \left(\frac{2}{x^3} \right) =$

(A) $-\frac{8}{x^4}$

(B) $-\frac{6}{x^4}$

(C) $-\frac{6}{x^2}$

(D) $\frac{6}{x^2}$

(E) $\frac{6}{x^4}$

4. If $f(x) = x^2 \ln x$, then $f'(x) =$

(A) 2

(B) x

(C) $2x \ln x$

(D) $2x \ln x + 2$

(E) $2x \ln x + x$

5. $\frac{d}{dx} (\sec^2 x) =$

(A) $-2 \sec^2 x \tan x$

(B) $-2 \sec^2 x$

(C) $-2 \sec x$

(D) $2 \sec x$

(E) $2 \sec^2 x \tan x$

6. An equation of the line tangent to the graph of f is $y + 8 = 3(x - 7)$. Which choice lists the correct properties of f ?

(A) $f(-7) = -8$ and $f'(-7) = 3$

(B) $f(-7) = 8$ and $f'(-7) = 3$

(C) $f(7) = -8$ and $f'(7) = -3$

(D) $f(7) = -8$ and $f'(7) = 3$

(E) $f(7) = 8$ and $f'(7) = 3$

7. $\frac{d}{dx}(e^4) =$

(A) 0

(B) $-e^4$

(C) $-4e^3$

(D) $4e^3$

(E) e^4

8. $\left. \frac{d}{dt}(\log_8 t) \right|_{t=8} =$

- (A) $\frac{1}{8 \ln 8}$ (B) $\frac{1}{\ln 8}$ (C) $\frac{1}{\log_8 e}$ (D) $\frac{1}{8}$ (E) 1

9. Given that f is differentiable at a , which of the following must be true?

- I. f is continuous at a .
- II. The graph of $y = f(x)$ does not contain a vertical tangent or sharp turn at $x = a$.
- III. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists.

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) II and III only
- (E) I, II, and III

10. If $g(x) = \sqrt{\sin 3x}$, then $g'\left(\frac{\pi}{6}\right)$ is

- (A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{2}$ (E) undefined

11. Which function is *not* differentiable over the set of all real numbers?

- (A) $|x + 3|$
(B) 6
(C) $\cos 2x$
(D) $5 + \sinh x$
(E) $x^4 - 2x^3 + 6$

12. What is an equation of the line tangent to the graph of $y = e^{2x}$ at $x = 2$?

(A) $y - e^4 = e^4(x - 2)$

(B) $y - e^4 = 2e^4(x - 2)$

(C) $y - e^4 = 2e^4(x + 2)$

(D) $y + e^4 = e^4(x - 2)$

(E) $y + e^4 = 2e^4(x + 2)$

13. It is known that $f(-1) = -3$ and $f'(-1) = 6$. If $a(x) = x^2 f(x)$, then $a'(-1) =$

(A) -15

(B) -12

(C) -6

(D) 6

(E) 12

14. The line $y = 4x + 8$ is parallel to the line tangent to the graph $y = x^2 - 4x - 4$ when

- (A) $x = -3$ (B) $x = -2$ (C) $x = 2$ (D) $x = 4$ (E) $x = 5$

15. For the curve $2x^2 + 3xy - y = 5$, $\frac{dy}{dx} =$

- (A) $\frac{3y - 4x}{1 - 3x}$ (B) $\frac{4x + 3y}{3x}$ (C) $\frac{4x - 3y}{1 - 3x}$ (D) $\frac{4x + 3y}{1 - 3x}$ (E) $\frac{4x + 3y}{1 + 3x}$

16. The slope of the line normal to the graph of $y = \sqrt{x}$ at $x = 9$ is

- (A) -6 (B) $-\frac{1}{6}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{6}$ (E) 6

17. If $y = \frac{1}{t^2 + 4}$, then $dy =$

- (A) $\frac{-2t}{t^2 + 4} dt$
(B) $\frac{-2t}{(t^2 + 4)^2} dt$
(C) $\frac{1}{t^2 + 4} dt$
(D) $\frac{2t}{t^2 + 4} dt$
(E) $\frac{2t}{(t^2 + 4)^2} dt$

18. $\sinh(\ln 2) =$

- (A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) 1 (E) 2

19. If $y = \sin^6 5x$, then $\frac{dy}{dx} =$

- (A) $6\sin^5 5x$
(B) $6\sin^5 5x \cos 5x$
(C) $30\sin^5 5x$
(D) $30\sin^5 5x \cos 5x$
(E) $30\sin^6 5x \cos 5x$

20. A linearization of $\sqrt{x}$ centered at $x = 1$ approximates $\sqrt{1.4}$ to be

- (A) 0.6 (B) 0.8 (C) 1 (D) 1.2 (E) 1.4

21. If $v(x) = x^2\sqrt{x}\sin x$, then $v'(x) =$

- (A) $\frac{5}{2}x\sqrt{x}\cos x$
(B) $2x\sqrt{x}\cos x$
(C) $-\frac{5}{2}x\sqrt{x}\sin x - x^2\sqrt{x}\cos x$
(D) $\frac{5}{2}x\sqrt{x}\sin x - x^2\sqrt{x}\cos x$
(E) $\frac{5}{2}x\sqrt{x}\sin x + x^2\sqrt{x}\cos x$

22. A cube's side lengths are all 1 inch. By using differentials, how much does the cube's volume (in cubic inches) *decrease* when its side lengths are all shrunk to 0.9 inch?

- (A) 0.1 (B) 0.2 (C) 0.3 (D) 0.5 (E) 0.6

23. If $y = \sin^{-1}(x^2 - 4)$, then $\frac{dy}{dx} =$

- (A) $\frac{-1}{\sqrt{1 - (x^2 - 4)^2}}$
- (B) $\frac{1}{\sqrt{1 - (x^2 - 4)^2}}$
- (C) $\frac{-2x}{\sqrt{1 - (x^2 - 4)^2}}$
- (D) $\frac{2x}{\sqrt{1 - (x^2 - 4)^2}}$
- (E) $\frac{2x}{\sqrt{1 + (x^2 - 4)^2}}$

24. $\lim_{h \rightarrow 0} \frac{\ln(1+h)}{h}$ is

- (A) -1 (B) 0 (C) 1 (D) e (E) nonexistent

25. If $y = \tan^{-1} 3x$, then $y'' =$

- (A) $\frac{-54x}{\sqrt{1-9x^2}}$ (B) $\frac{-54x}{9x^2-1}$ (C) $\frac{-54x}{(1-9x^2)^2}$ (D) $\frac{-54x}{(1+9x^2)^2}$ (E) $\frac{54x}{(1+9x^2)^2}$

26. If f and g are inverse functions of each other, with $f(2) = 4$ and $f'(2) = 7$, then $g'(4) =$

- (A) $-\frac{1}{7}$ (B) $\frac{1}{7}$ (C) $\frac{1}{4}$ (D) 4 (E) 7

27. If $y = 8^x$, then $y' =$

- (A) 8^x (B) $8^x \ln 8$ (C) $8^x \log_8 e$ (D) $e^x \ln 8$ (E) $e^x \log_8 e$

28. $\frac{d}{dx} \ln(x^2 + 4x - 8) =$

(A) $\frac{1}{2x+4}$

(B) $\frac{1}{x^2 + 4x - 8}$

(C) $\frac{-2x-4}{x^2 + 4x - 8}$

(D) $\frac{2x+4}{x^2 + 4x - 8}$

(E) $\frac{2x+4}{\ln(x^2 + 4x - 8)}$

29. The range of $4 \operatorname{sech} 2x + 5$ is

(A) $(0, 4]$

(B) $(0, 5]$

(C) $(4, 5]$

(D) $(4, 9]$

(E) $(5, 9]$

30. If $f(x) = \cos x$, then $f^{(66)}(x) =$

- (A) $-\cos x$ (B) $-\sin x$ (C) $\cos x$ (D) $\sin x$ (E) $\tan x$

31. If $e^{2x} - \sin y = y^2$, then $\frac{dy}{dx} =$

- (A) $\frac{e^{2x}}{2y + \cos y}$
(B) $\frac{e^{2x}}{2y - \cos y}$
(C) $\frac{2e^{2x}}{2y - \cos y}$
(D) $\frac{2e^{2x}}{\cos y - 2y}$
(E) $\frac{2e^{2x}}{2y + \cos y}$

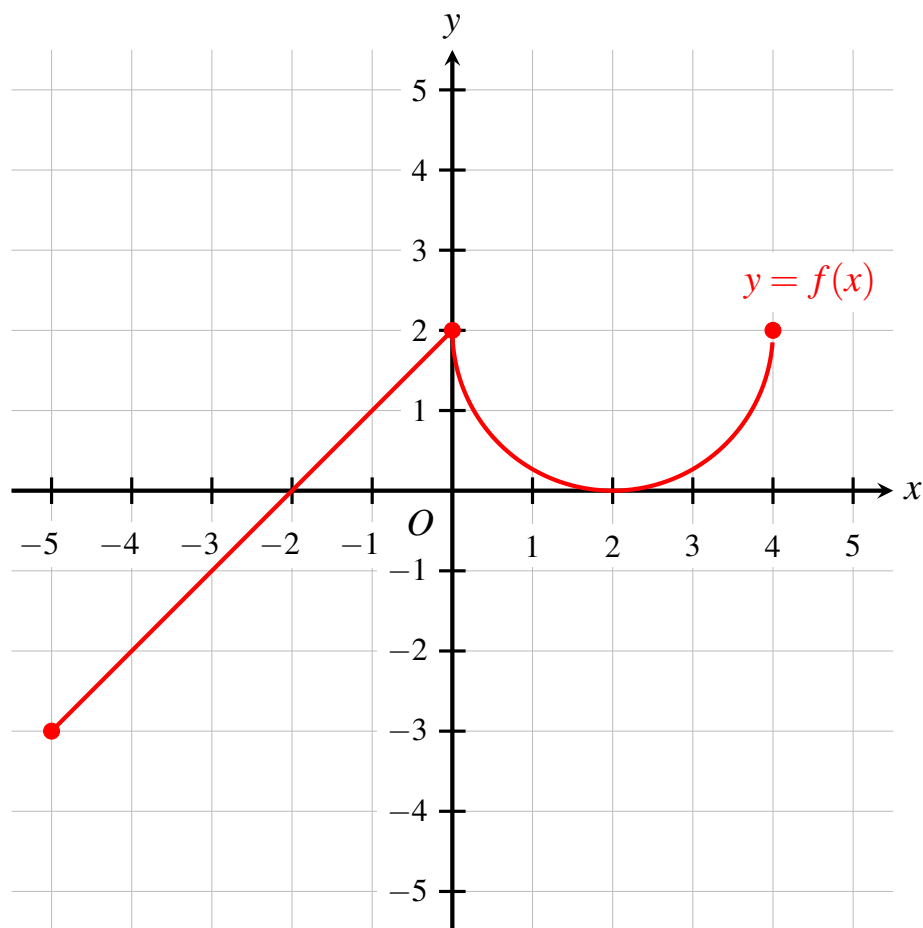
32. $\lim_{x \rightarrow \pi/3} \frac{\frac{1}{2} - \cos x}{x - \frac{\pi}{3}}$ is

- (A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

33. $\frac{d}{dx} \left(\frac{\tan x}{x^2} \right) =$

- (A) $\frac{\sec^2 x}{2x}$
(B) $\frac{2x \tan x - x^2 \sec^2 x}{x^2}$
(C) $\frac{2x \tan x - x^2 \sec^2 x}{x^4}$
(D) $\frac{x^2 \sec^2 x - 2x \tan x}{x^2}$
(E) $\frac{x^2 \sec^2 x - 2x \tan x}{x^4}$

Questions 34–36 refer to the following graph.



34. At which value of x is f not differentiable?

- (A) -4 (B) -2 (C) 0 (D) 2 (E) 3

35. $\lim_{x \rightarrow -5^+} \frac{f(x) + 3}{x + 5}$ is

- (A) -5 (B) -3 (C) 1 (D) 3 (E) nonexistent

36. Let h be a differentiable function such that $h(-2) = 2$ and $h'(-2) = 3$. If $p(x) = \frac{f(2x)}{3h(x)}$, then $p'(-2) =$

- (A) $-\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{5}{3}$

37. A particle travels along the x -axis with time t according to the function $x(t) = \ln(t + 1)$. At $t = 2$, the particle's acceleration is

- (A) $-\frac{1}{9}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{27}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

38. $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$ is

- (A) e^{-2} (B) e^{-1} (C) e (D) e^2 (E) nonexistent

39. A circle's radius is increased from 10 to 10.1. By using differentials, the change in the circle's area is approximately

- (A) 2π (B) 4π (C) 10π (D) 20π (E) 40π

40. At which values of x does $y = \frac{1}{3}x^3 + 2x^2 - 5x + 6$ have a horizontal tangent?

I. $x = -5$

II. $x = -1$

III. $x = 1$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

41. $\frac{d}{dx} \left(\frac{\sin^2 x}{e^{4x}} \right) =$

(A) $\sin 2x - 4 \sin^2 x$

(B) $\frac{\sin 2x + 4 \sin^2 x}{e^{4x}}$

(C) $\frac{\sin 2x - 4 \sin^2 x}{e^{4x}}$

(D) $\frac{\sin 2x + 4 \sin^2 x}{e^{8x}}$

(E) $\frac{\sin 2x - 4 \sin^2 x}{e^{8x}}$

42. $\frac{d}{dx} \tanh^{-1}(\cos x) =$

(A) $-\csc x$

(B) $-\sec x$

(C) $\cos x$

(D) $\csc x$

(E) $\sec x$

43. For the curve $y^3 - y = x + 8$, $\frac{d^2y}{dx^2} =$

(A) $-\frac{6y}{(3y^2 - 1)^2}$

(B) $\frac{6y}{(3y^2 - 1)^2}$

(C) $-\frac{6y}{(3y^2 - 1)^3}$

(D) $\frac{6y}{(3y^2 - 1)^3}$

(E) $-\frac{6y}{(3y^2 - 1)^4}$

44. $\frac{d}{dx}(x^{\sec x}) =$

(A) $(\sec x)x^{\sec x-1}$

(B) $(\sec x)x^{\sec x}$

(C) $(\sec x \tan x)x^{\sec x}$

(D) $\left(\frac{\sec x}{x} - \sec x \tan x \ln x\right)x^{\sec x}$

(E) $\left(\frac{\sec x}{x} + \sec x \tan x \ln x\right)x^{\sec x}$

45. The curve $2x^3y + 16x^2y^2 + 1 = 0$ has a vertical tangent when

I. $x = -2$

II. $x = 0$

III. $x = 2$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II, and III

46. If $y = \sin^4(\sqrt{x})$, then $y' =$

(A) $4\sin^3(\sqrt{x})$

(B) $4\sin^3(\sqrt{x})\cos(\sqrt{x})$

(C) $\frac{2\sin^3(\sqrt{x})}{\sqrt{x}}$

(D) $\frac{2\sin^3(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}$

(E) $\frac{2\cos^3(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}$

47. $\frac{d}{dx}(x^4 e^{2x} \sec x) =$

(A) $4x^3 e^{2x} \sec x + x^4 e^{2x} \sec x + x^4 e^{2x} \sec x$

(B) $4x^3 e^{2x} \sec x + 2x^4 e^{2x} \sec x + x^4 e^{2x} \sec x$

(C) $4x^3 e^{2x} \sec x + x^4 e^{2x} \sec x + x^4 e^{2x} \sec x \tan x$

(D) $4x^3 e^{2x} \sec x + 2x^4 e^{2x} \sec x + x^4 e^{2x} \sec x \tan x$

(E) $x^4 e^{2x} \sec x + 2x^4 e^{2x} \sec x + x^4 e^{2x} \sec x$

48. $\frac{d}{dx} \left(\frac{e^{\sin x} \tan x}{x^5 \sin 5x} \right) =$

(A) $\frac{e^{2\sin x} \tan x + e^{\sin x} \tan x \sec^2 x - 5x^4 e^{\sin x} - 5e^{\sin x} \cos 5x}{x^5 \sin 5x}$

(B) $\frac{5x^4 e^{\sin x} + 5e^{\sin x} \cos 5x - e^{2\sin x} \tan x - e^{\sin x} \tan x \sec^2 x}{x^5 \sin 5x}$

(C) $\frac{xe^{\sin x} \sin x + xe^{\sin x} \sec^2 x - 5e^{\sin x} \tan x - 5xe^{\sin x} \tan x \cot 5x}{x^6 \sin 5x}$

(D) $\frac{5e^{\sin x} \tan x + 5xe^{\sin x} \tan x \cot 5x - xe^{\sin x} \sin x - xe^{\sin x} \sec^2 x}{x^6 \sin 5x}$

(E) $\frac{e^{2\sin x} \tan x + e^{\sin x} \tan x \sec^2 x - 5x^4 e^{\sin x} - 5e^{\sin x} \cos 5x}{x^{10} \sin^2 5x}$

49. A projectile's height, in feet, above the ground as a function of time, in seconds, is given by the function $h(t) = -16t^2 - 64t + 80$. The projectile's speed, in feet per second, upon striking the ground is

- (A) 32 (B) 64 (C) 96 (D) 128 (E) 192

50. $\frac{d}{dx} \left(\frac{1}{\ln(\ln(\ln x))} \right) =$

(A) $\frac{-1}{\ln(\ln x) [\ln(\ln(\ln x))]^2}$

(B) $\frac{1}{x(\ln x) \ln(\ln x) \ln(\ln(\ln x))}$

(C) $\frac{-1}{x(\ln x) \ln(\ln x) \ln(\ln(\ln x))}$

(D) $\frac{1}{x(\ln x) \ln(\ln x) [\ln(\ln(\ln x))]^2}$

(E) $\frac{-1}{x(\ln x) \ln(\ln x) [\ln(\ln(\ln x))]^2}$

This marks the end of the test. The following page contains the answers to all the questions.

- | | |
|-------|-------|
| 1. D | 33. E |
| 2. C | 34. C |
| 3. B | 35. C |
| 4. E | 36. D |
| 5. E | 37. A |
| 6. D | 38. D |
| 7. A | 39. A |
| 8. A | 40. C |
| 9. E | 41. C |
| 10. B | 42. A |
| 11. A | 43. C |
| 12. B | 44. E |
| 13. E | 45. C |
| 14. D | 46. D |
| 15. D | 47. D |
| 16. A | 48. C |
| 17. B | 49. C |
| 18. B | 50. E |
| 19. D | |
| 20. D | |
| 21. E | |
| 22. C | |
| 23. D | |
| 24. C | |
| 25. D | |
| 26. B | |
| 27. B | |
| 28. D | |
| 29. E | |
| 30. A | |
| 31. E | |
| 32. D | |